AKTU B.Tech Sem-3

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit -3 : Lec - 1

GW
GATEWAY CLASSES

Topics to be Covered

- Magnetostatics
- Magnetic Field Intensity
- Magnetic Flux density
- Biot Savart's Law
- PYQs

By Swati Ma'am

- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

GATEWAY CLASSES

Udaan Batch 2.0

B.Tech II Year

EE & EE Allied

Electromagnetic Field Theory (BEE301)

Unit-3 Syllabus

Magneto statics : Magneto-static fields, Biot - Savart's Law, Ampere's circuit law, Maxwell's equation, Application of ampere's law, Magnetic flux density- Maxwell's equation, Maxwell's equation for static fields, magnetic scalar and vector potential.

MAGNETIC FIELD INTENSITY

Stationary charges produce Electric field.

In 1820, Oersted discovered that a compass needle suffers a deflection when brought near a current carrying wire.

This shows that an electric charge in motion produces a magnetic field as well as electric field in the space around it.

Two Laws govern Magnetostatics :- Biot-Savart law
and
Ampere's law

Electrostatics	Magnetostatics
E – Electric Field Intensity	H – Magnetic Field Intensity
D – Electric Flux Density or Electric Displacement	B – Magnetic Flux Density or Magnetic Induction
ε – Permittivity of medium	μ – Permeability of medium
$D = \varepsilon E$	$B = \mu H$
where $\varepsilon = \varepsilon_o \varepsilon_r$ ε is permittivity of medium ε_0 is permittivity of free space (8.85×10^{-12} F/m), Also called absolute permittivity ε_r is relative permittivity For free space $\varepsilon_r = 1$, $D = \varepsilon_0 E$	where $\mu = \mu_o \mu_r$ μ is permeability of medium μ_0 is permeability of free space ($4\pi \times 10^{-7}$ H/m) Also called absolute permeability μ_r is relative permeability For free space $\mu_r = 1$, therefore $B = \mu_0 H$

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MAGNETIC FLUX DENSITY B

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The magnetic flux density B is a measure of magnetic flux produced by magnetic field.

B is defined as the magnetic flux lines passing per unit area through a plane at right angles to the flux. Therefore,

Magnetic flux density = total flux / surface area

or

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Magnetic Flux Density Equation

$$B = \frac{\phi}{S}$$

weber/metre^2 or tesla.

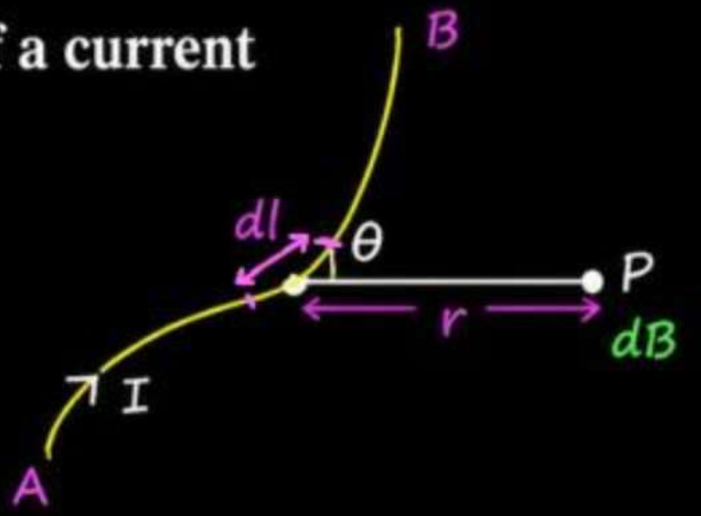
The total magnetic flux passing through the surface is given by

$$\vec{\phi}_S = \vec{B} \cdot \vec{dS}$$

BIOT- SAVART'S LAW

Biot and Savart observed that dB at point P due to small element dl of a current carrying conductor depends upon the following factors:

(i) $dB \propto I$ (ii) $dB \propto dl$ (iii) $dB \propto \sin\theta$ (iv) $dB \propto 1/r^2$



Combining all these factors,

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

When the conductor is placed in vacuum or air, then

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

where μ_0 is the permeability of free space.

$(\mu_0 / 4\pi)$ is proportionality constant whose value is 10^{-7} weber/(amp-metre).

The magnetic field intensity H is given by

$$dH = \frac{1}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$\vec{dl} \times \hat{a}_r$

$|\vec{dl}||\hat{a}_r|\sin\theta$

$dl \cdot 1 \cdot \sin\theta$

In Vector Form

The magnetic field intensity H in vector form is given by

$$d\vec{H} = \frac{1}{4\pi} \frac{I(d\vec{l} \times \hat{a}_r)}{r^2}$$

The magnetic induction B is vector form is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \hat{a}_r)}{r^2}$$

where $\hat{a}_r$ is unit vector along the direction of $\vec{r}$.

As $\hat{a}_r = \frac{\vec{r}}{|\vec{r}|}$, we have the value of $d\vec{B}$ in free space as

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

The resultant field can be obtained by integrating above equation $\vec{B} = \int d\vec{B}$

The direction of $\vec{B}$ is given by Right Hand Screw Rule. With the screw placed in the direction of current flow, the direction of advance of the screw is the direction of $d\vec{B}$.

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Different Forms

(i) Biot Savart's Law in terms of magnetic field intensity H (ampere/metre)

$$\mu_0 = \frac{Magnetic\ flux\ density}{Magnetic\ field\ intensity} = \frac{\vec{B}}{\vec{H}}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$d\vec{H} = \frac{1}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

(ii) Biot Savart's Law in terms of Current Density

The current density $\vec{J}$ at a point on the element $d\vec{l}$, area of cross-section dA and volume dV is given by

$$I\, dl = \vec{J}\, dA\, dl = \vec{J}\, dV$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J} \times \vec{r}}{r^3} dV$$

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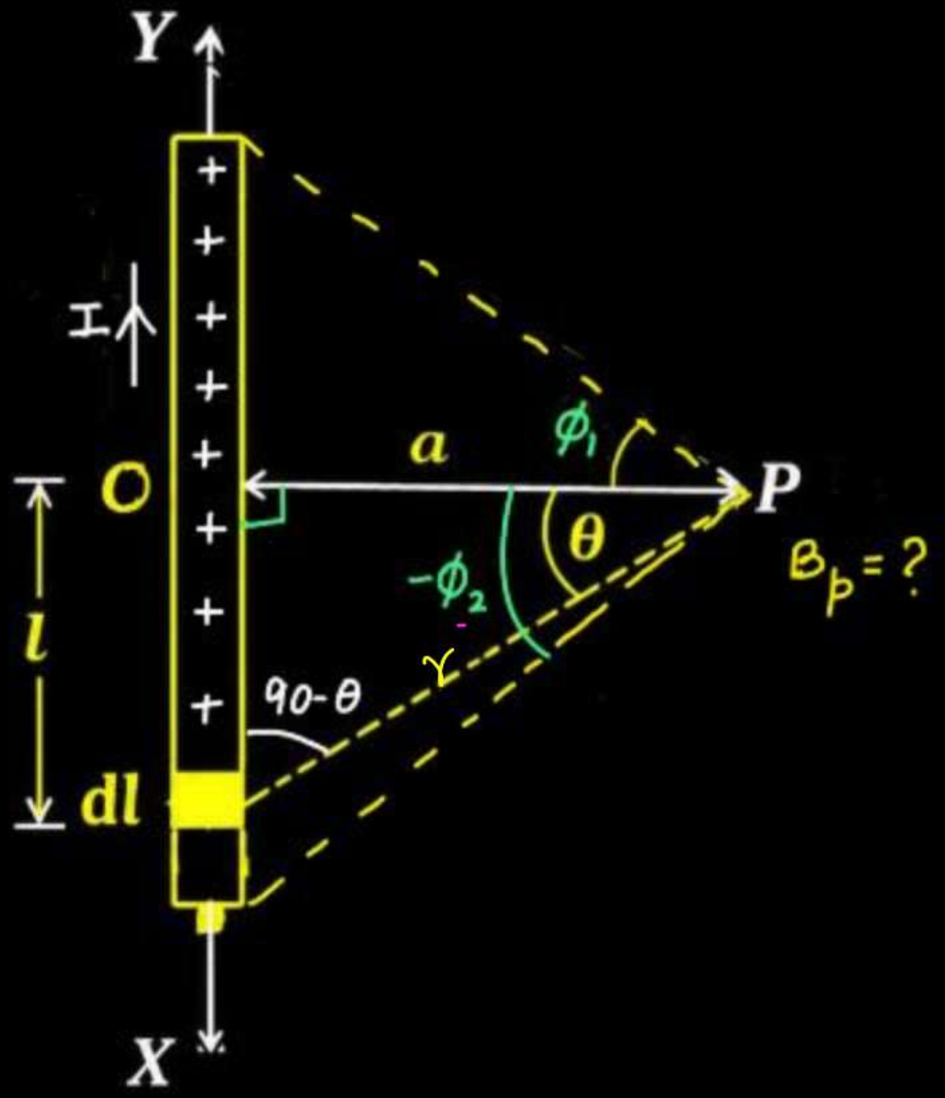
Magnetic Field Intensity due to a Line Charge:-

(i) Finite Line Charge

(ii) Infinite Line Charge

(iii) Semi-finite Line Charge



(i)

MAGNETIC FIELD INTENSITY DUE TO FINITE LONG CONDUCTOR CARRYING A CURRENT *I*

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Let the wire lie along XY axis, *I* is the current flowing through it.

At point P, B_P = ?

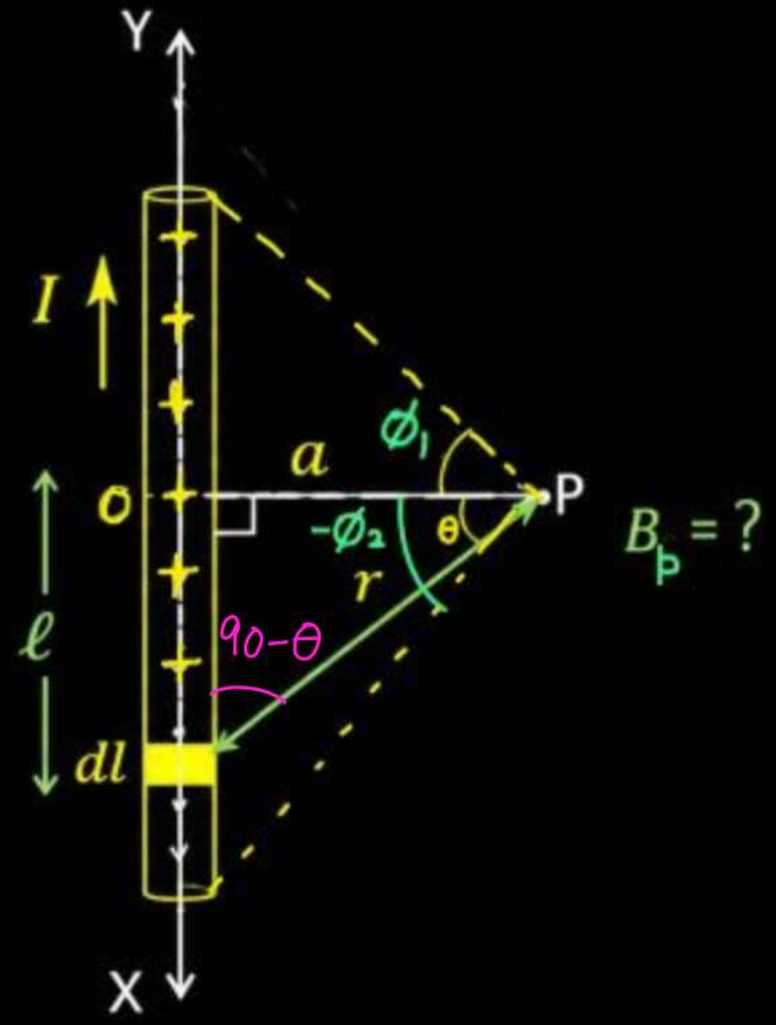
Drop a perpendicular from P

Distance of point P from wire = a

Due to element dl

$$dB = \frac{\mu_0}{4\pi} \frac{I\ dl \sin(90-\theta)}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I\ dl\ \cos\theta}{r^2}$$

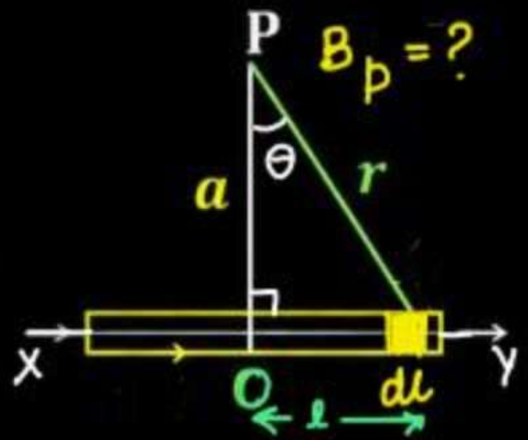


$B = \int dB$

ϕ_1 → Note

$$B = \int_{-\phi_2}^{\phi_1} \frac{\mu_0}{4\pi} \frac{I\, dl\, \cos\theta}{r^2} \quad -(1)$$

$-\phi_2$ → Note



From *fig.*, $\cos\theta = \frac{a}{r}$

$$r = a \sec\theta \quad -(2)$$

Also $\tan\theta = \frac{l}{a}$

$$l = a \tan\theta$$

$$dl = a \sec^2\theta\, d\theta \quad (3)$$

Put (3) & (2) in (1)

$$B = \int_{-\phi_2}^{\phi_1} \frac{\mu_0}{4\pi} \frac{I \cos\theta\, (a \sec^2\theta\, d\theta)}{a^2 \sec^2\theta}$$

$$B = \int_{-\phi_2}^{\phi_1} \frac{\mu_0\, I}{4\pi\, a} \cos\theta\, d\theta$$

$$= \frac{\mu_0\, I}{4\pi\, a} \left[\sin\theta\right]_{-\phi_2}^{\phi_1}$$

Learn

$$\vec{B} = \frac{\mu_0\, I}{4\pi\, a} \left[\sin\phi_1 + \sin\phi_2\right]$$

(ii) MAGNETIC FIELD INTENSITY DUE TO INFINITELY LONG CONDUCTOR CARRYING A CURRENT I

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Note :- Do derivation (i) first , then

We Know,

$$B = \frac{\mu_0 I}{4\pi a}\left[\sin\phi_1 + \sin\phi_2\right]$$

Put $\phi_1 = 90^\circ$, $\phi_2 = 90^\circ$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi a}\left[\sin 90^\circ + \sin 90^\circ\right]$$

$$B = \frac{\mu_0 I}{4\pi a}\left[1 + 1\right]$$

$$B = \frac{\mu_0 I}{4\pi a} \times 2$$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$

(iii)

MAGNETIC FIELD INTENSITY DUE TO SEMI- FINITE LONG CONDUCTOR CARRYING A CURRENT I

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Note:- Do derivation (i) & (ii) first, then

We know ,for an infinite long current carrying conductor , Magnetic Flux density is given by

$$B = \frac{\mu_0 I}{2\pi a}$$

Learn

For semi finite long current carrying conductor ,whose length is from 0 to ∞ or from -∞ to 0

$$B = \frac{\mu_0 I}{4\pi a}$$

Learn

Que) A current of 10 A is flowing from origin to negative infinity along x-axis. Find the magnetic field produced by it at (−2, 3, 0).

Wire lies on negative x-axis, from x=0 to x=−∞ , For a semi-infinite straight wire, the magnetic field magnitude is:

$$B = \frac{\mu_0}{4\pi}\frac{I}{a}$$

where, $\mu_0 = 4\pi \times 10^{-7}$

$I = 10\ A$

$$a = \sqrt{y^2 + z^2} = \sqrt{3^2 + 0^2} = 3$$

$$B = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{10}{3}$$

$$B = 3.33 \times 10^{-7}\ Tesla\ Ans$$

Direction of B

Using Right Hand Rule,

Thumb → direction of current (towards −X axis)

Fingers curl → direction of magnetic field.

Point (−2, 3, 0) is above Y axis

$\therefore \vec{B}$ is along $+Z$ axis.

Magnetic Field Intensity due to a Circular Loop

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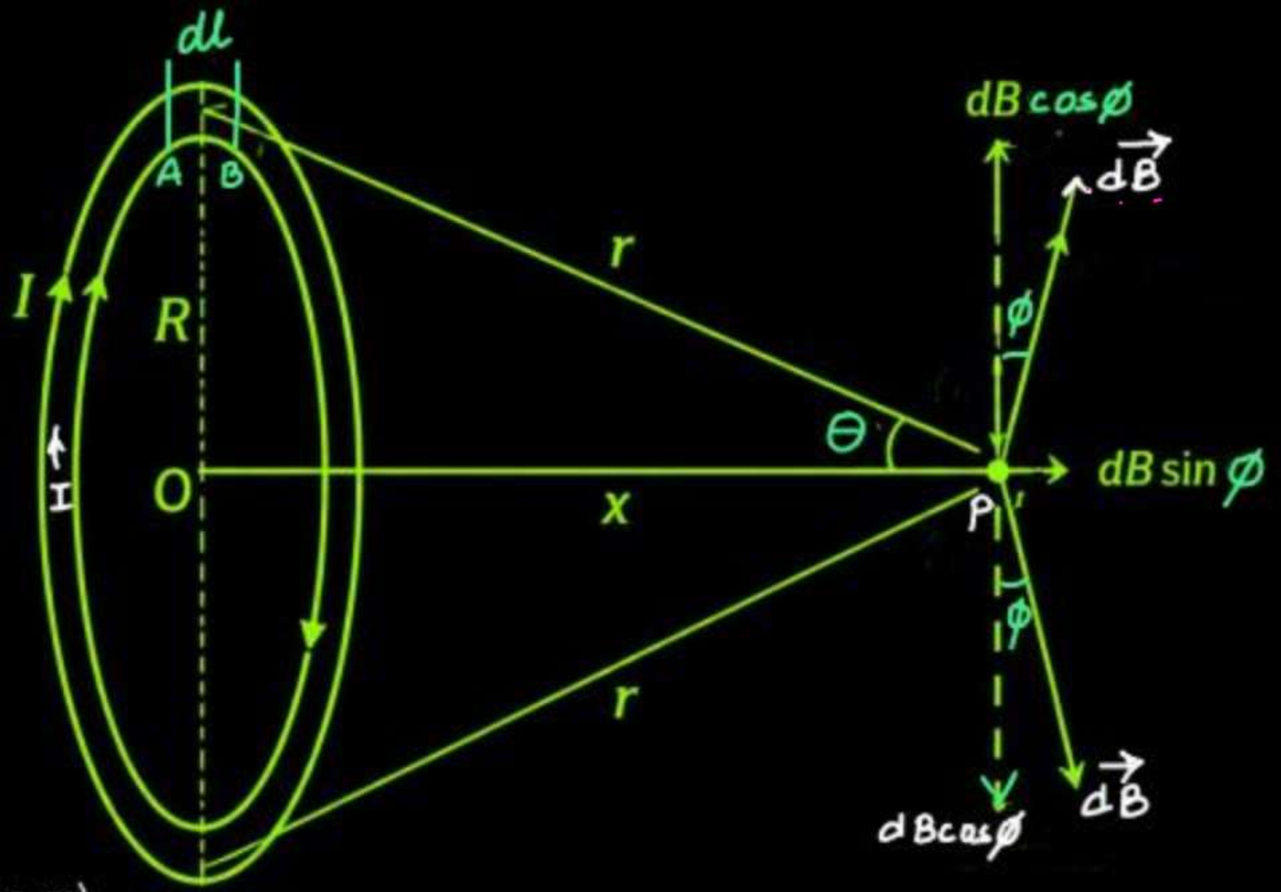
The magnetic field intensity (B) at any point on the axis of a current-carrying circular loop is determined using the Biot-Savart Law.

It is directly proportional to the current and the square of the radius, and inversely proportional to the distance from the loop.

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$



Where:

- B = Magnetic field intensity (in Tesla, T)
- μ_0 = Permeability of free space ($4\pi \times 10^{-7}$ T·m/Wb)
- I = Current flowing through the loop (in Amperes, A)
- R = Radius of the circular loop (in metres, m)
- x = Distance along the axis from the center of the loop (in metres, m)

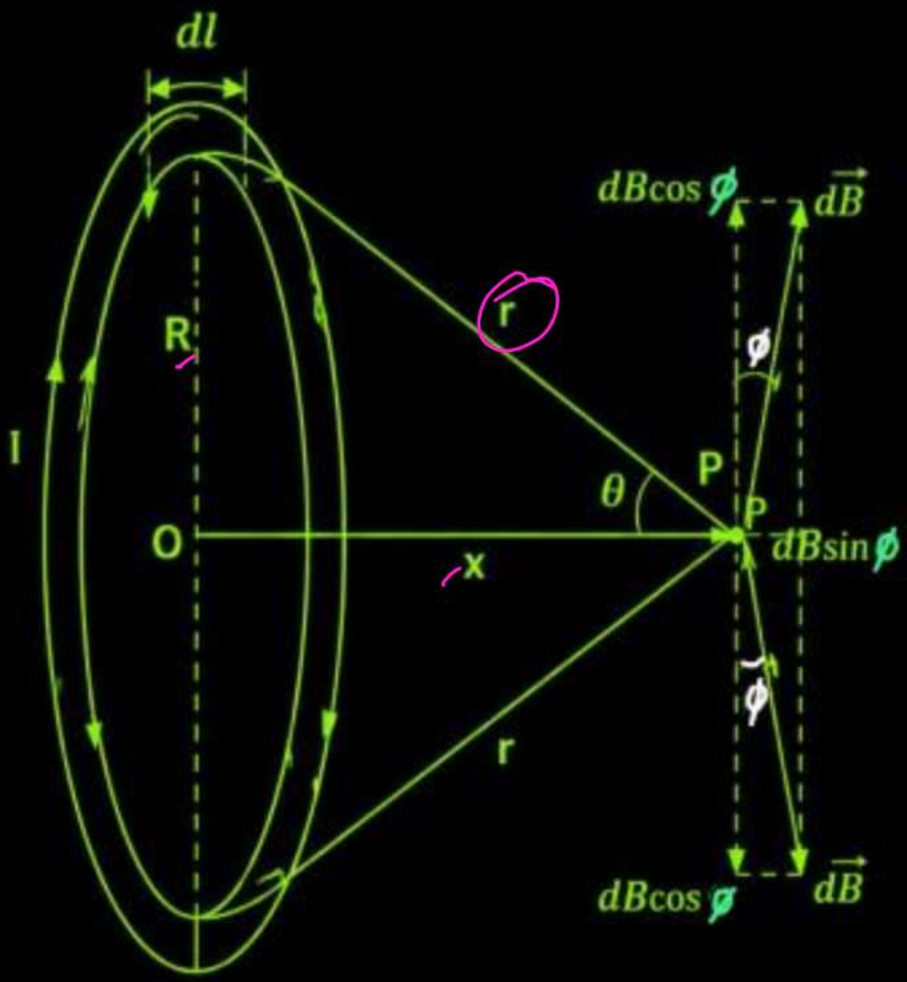
Derivation

1. Define Current Element and Distance

Consider a circular loop of radius R carrying current I lying in the yz-plane, with its center at the origin O. We need to find the magnetic field at point P on the x-axis at a distance x from the center.

Consider a small current element $d\vec{l}$ on the loop. The distance r from this element to point P is

$$r = \sqrt{R^2 + x^2}$$



2. Apply the Biot–Savart Law

The magnetic field $d\vec{B}$ due to the current element $d\vec{l}$ at point P is given by the Biot–Savart law.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^2}$$

Since the current element $d\vec{l}$ is perpendicular to the position vector $\vec{r}$, the magnitude of $d\vec{B}$ is

↳ $\because \sin 90 = 1$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl}{r^2}$$

(The direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\vec{r}$.)

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3. Resolve Components

The field $d\vec{B}$ can be resolved into two rectangular components:

- An axial component $dB_x = dB\sin\theta$
- A perpendicular component $dB_\perp = dB\cos\theta$

4. Utilize Symmetry

By symmetry, for every current element, there is a diametrically opposite element that produces an equal and opposite perpendicular component ($dB_\perp$). These perpendicular components cancel each other out. Thus, only the axial components (dB_x) add up to form the total magnetic field.

5. Integrate Over the Loop

Summing all the axial components dB_x over the entire loop:

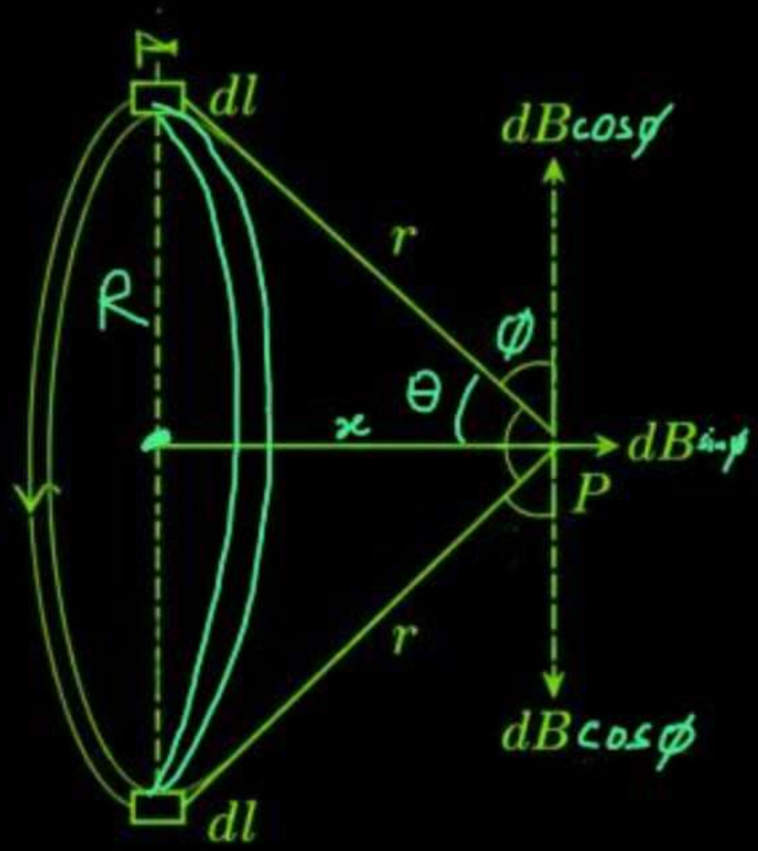
$$B = \int dB_x = \int dB\sin\theta$$

From the geometry of the loop:

$$\sin\theta = \frac{R}{r} = \frac{R}{\sqrt{R^2 + x^2}}$$

Substituting the values of dB and $\sin\theta$,

$$B = \int \left(\frac{\mu_0}{4\pi}\frac{I\,dl}{R^2 + x^2}\right)\left(\frac{R}{\sqrt{R^2 + x^2}}\right)$$



Since μ_0, I, R, and x are constant for all elements on the loop, they can be taken out of the integral:

$$B = \frac{\mu_0 I R}{4\pi (R^2 + x^2)^{3/2}} \int dl$$

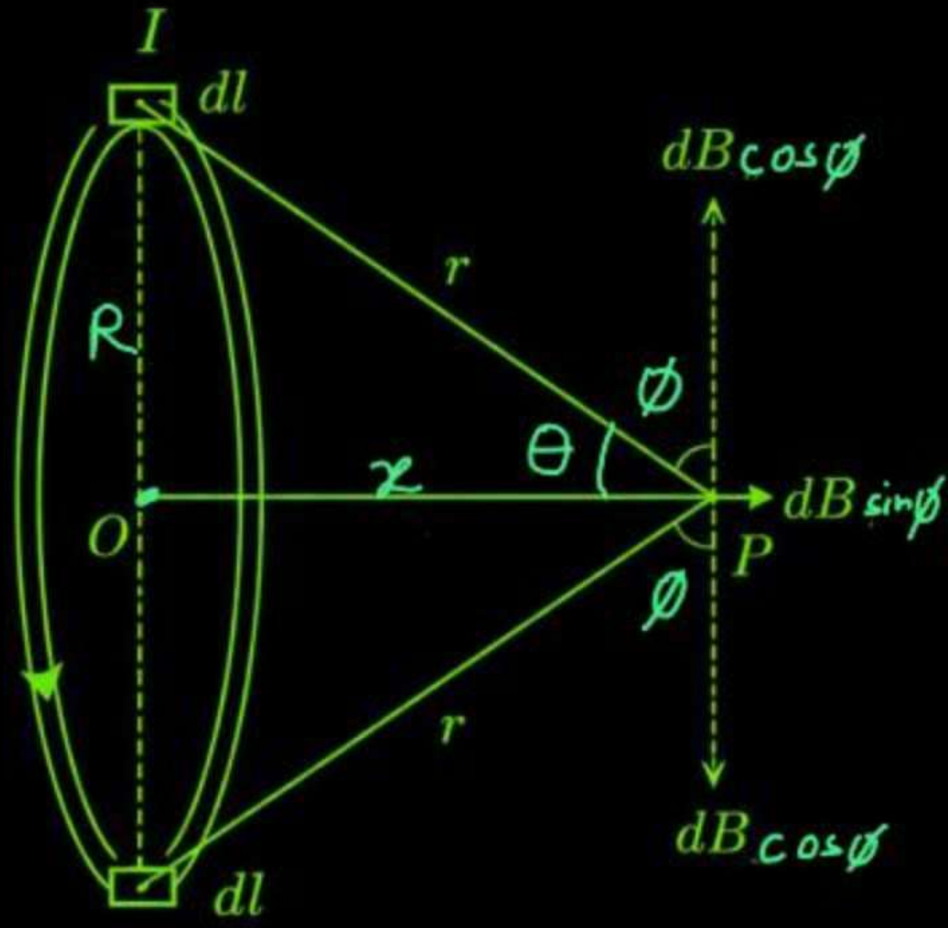
The integral of dl ($\int dl$) over the entire circular loop is its circumference, $2\pi R$.

Therefore:

$$B = \frac{\mu_0 I R}{4\pi (R^2 + x^2)^{3/2}} (2\pi R)$$

Simplifying the equation gives the final result:

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$



$\because \quad B = \mu_0 H$

$$\therefore \quad H = \frac{I R^2}{2(R^2 + x^2)^{3/2}}$$

A circular loop located on $x^2 + y^2 = 9,\ z = 0$ carries a direct current of 10A along a_ϕ. Determine H at $(0,0,4)$ and $(0,0,-4)$.

Eqⁿ of circle $x^2 + y^2 = R^2 = 9$
∴ Radius of circle = 3

Given:

- Circular loop: $x^2 + y^2 = 9,\ z = 0$
- Radius: $R = 3\ m$
- Current: $I = 10\ A$
- Current direction: $+a_\phi$ (counterclockwise when viewed from $+z$)
- Find H at:
  1. $(0, 0, 4)$
  2. $(0, 0, -4)$

Formula for magnetic field intensity on the axis of a circular loop

$$\vec{H} = \frac{IR^2}{2\,(R^2 + x^2)^{3/2}}\,\hat{a}_z$$

where x is the distance from the center along the axis.

Step 1: Calculate the magnitude

Here,

- $R = 3\ m$
- $x = 4\ m$
- $I = 10\ A$

$$H = \frac{I R^2}{2(R^2 + x^2)^{3/2}}$$

$$H = \frac{10(3)^2}{2(3^2 + 4^2)^{3/2}}$$

$$= \frac{90}{2(25)^{3/2}}$$

$$(25)^{3/2} = 125$$

$$H = \frac{90}{250} = 0.36\ \mathrm{A/m}$$

Step 2: Determine the direction

Using the **right-hand rule**:

- Current is along $+a_{\phi}$.
- The magnetic field points in the **+z-direction** everywhere on the axis of the loop (both above and below the loop).

Hence,

At (0, 0, 4)

$$\mathbf{H}(0,0,4) = 0.36\,\mathbf{a}_z \ \text{A/m}$$

At (0, 0, −4)

$$\mathbf{H}(0,0,-4) = 0.36\,\mathbf{a}_z \ \text{A/m}$$

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(ELECTROMAGNETIC FIELD THEORY)

Unit -3 : Lec - 2

Topics to be Covered

- Ampere's Circuital Law
- PYQs

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- M.Tech (Honours)
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Ampere's Circuital Law

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According to Ampere's law, the line integral of H along any closed path is exactly equal to the direct current enclosed by the path.

$$\oint \vec{H} \cdot \vec{dl} = I$$

closed path needs to be assumed to apply it.

where,

$\oint$ = closed path integration

H = Magnetic field intensity

dl = small differential length

I = current enclosed by the path

In terms of magnetic flux density, the law can be expressed as

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

where μ_0 is absolute permeability .

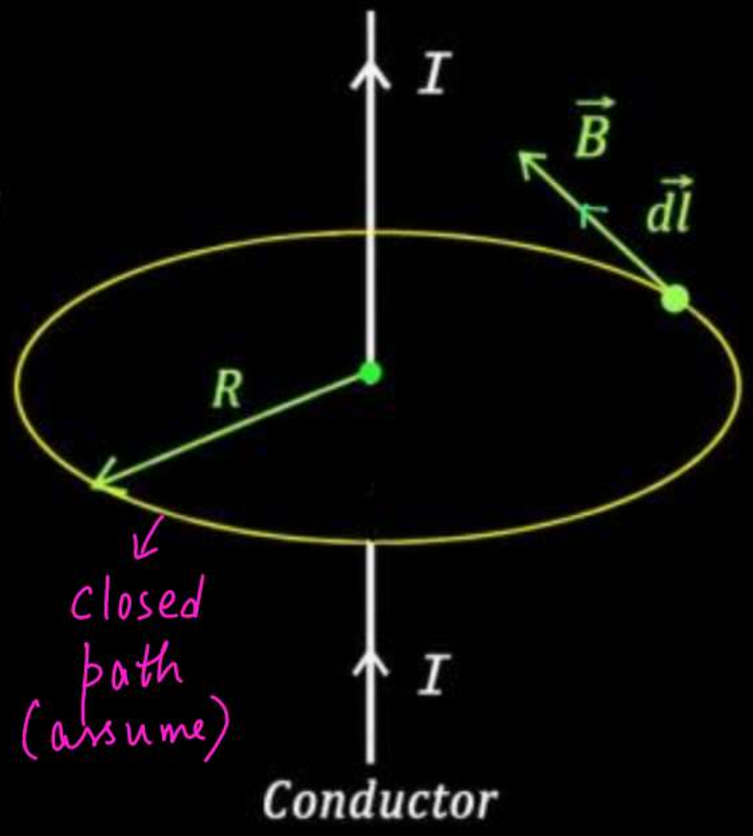
Proof of Ampere's Circuital Law

Consider a long straight conductor carrying a current I .

Let the conductor be perpendicular to the page directed outward.

The magnitude of magnetic induction at a distance R from the conductor is given by

$$B = \frac{\mu_0 I}{2\pi R} \quad -(1)$$ (Biot Savart Law)



The line integral of B and dl, which is given by

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos 0^0 = B \oint dl$$

$$= B\,(2\pi R) \quad -(2)$$

where $\oint dl = 2\pi R$ = circumference of the circle.

Substituting the value of B from eq. (1) in eq. (2), we get

$$\oint \vec{B} \cdot \vec{dl} = \frac{\mu_0 I}{2\pi R} \times 2\pi R = \mu_0 I$$

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$$

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Ampere's Law in Integral Form

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$$\oint \vec{H} \cdot d\vec{l} = \oint \vec{J} \cdot d\vec{s} = I$$

Ampere's Law in Differential Form

According to Ampere's Law,

$$\oint_c \vec{H} \cdot d\vec{l} = \oint_{\Delta s} \vec{J} \cdot d\vec{s}$$

Dividing both sides by Δs when $\Delta s \to 0$, we have

$$\lim_{\Delta s \to 0} \frac{\oint_c \vec{H} \cdot d\vec{l}}{\Delta s} = \lim_{\Delta s \to 0} \frac{\oint_{\Delta s} \vec{J} \cdot d\vec{s}}{\Delta s}$$

$$= \frac{J\ \hat{a}_n\ \Delta s}{\Delta s} = J\ \hat{a}_n = \vec{J}$$

As per Stokes Theorem, $\oint \vec{H} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{H}) d\vec{s}$

$\vec{\nabla} \times \vec{H}$ = curl $\vec{H}$

$$curl\ \vec{H} = \vec{J}$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}}$$

Ampere's Circuital Law in Differential Form.

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APPLICATIONS OF AMPERE'S CIRCUITAL LAW

By Using Ampere's Circuital Law one can find Magnetic Field Intensity due to

(i) INFINITELY LONG CONDUCTOR CARRYING A CURRENT I (using → Biot-Savart's Law, → Ampere's Circuital Law)

(ii) LONG HOLLOW CYLINDER (COAXIAL CABLE) CARRYING A CURRENT I

(iii) UNIFORM SHEET OF CHARGE

MAGNETIC FIELD INTENSITY DUE TO INFINITELY LONG CONDUCTOR CARRYING A CURRENT I

Applying Ampere's circuital law to this closed circuit , we have

$$\oint \vec{H} \cdot d\vec{\ell} = I$$

$$H \oint dl = I$$

$$H \left[2\pi\rho \right] = I$$

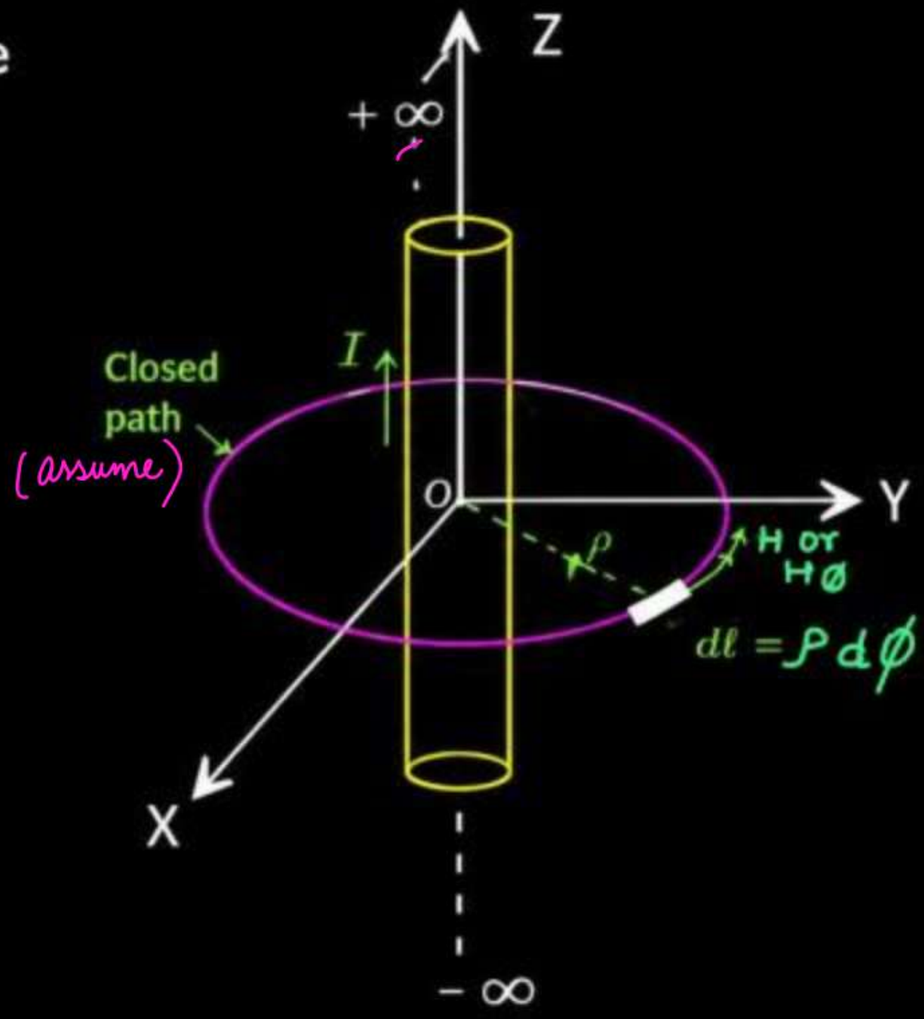
$$H = \frac{I}{2\pi\rho}$$

Hence , the magnetic field intensity in vector form is

$$\vec{H} = \frac{I}{2\pi\rho} \cdot \hat{a}_\phi \quad A/m$$

The magnetic flux density is given by

$$\vec{B} = \mu \vec{H} = \frac{\mu I}{2\pi\rho} \hat{a}_\phi \quad tesla$$



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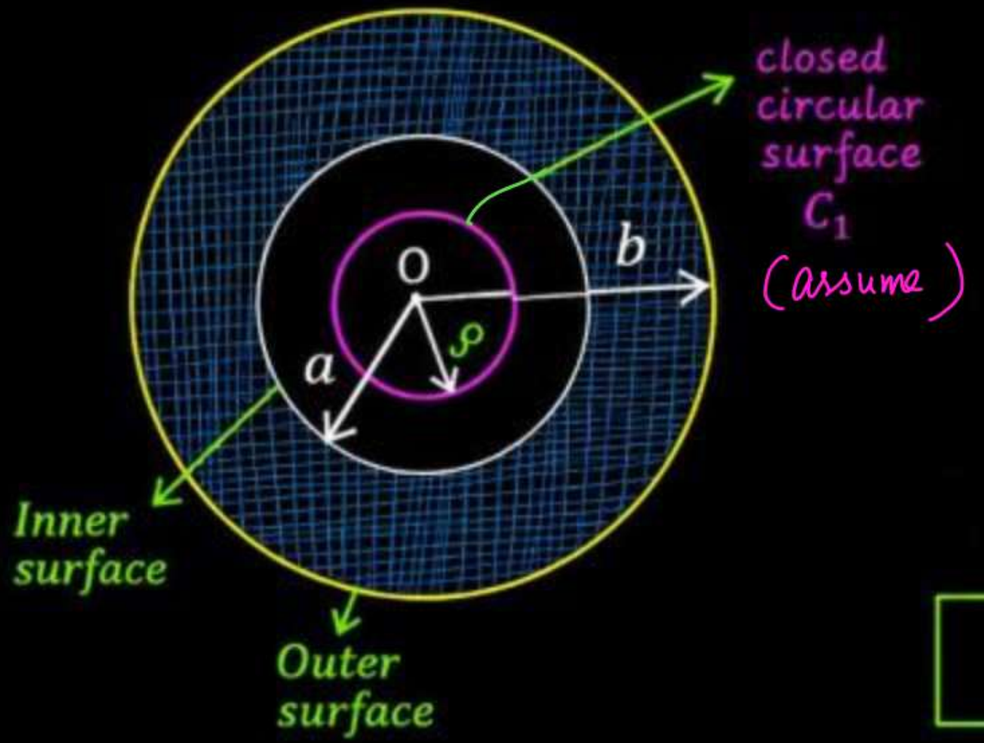
MAGNETIC FIELD INTENSITY DUE TO LONG HOLLOW CYLINDER (COAXIAL CABLE) CARRYING A CURRENT I

(i) *For* $\rho \leq a$



Applying Ampère's Circuital Law,

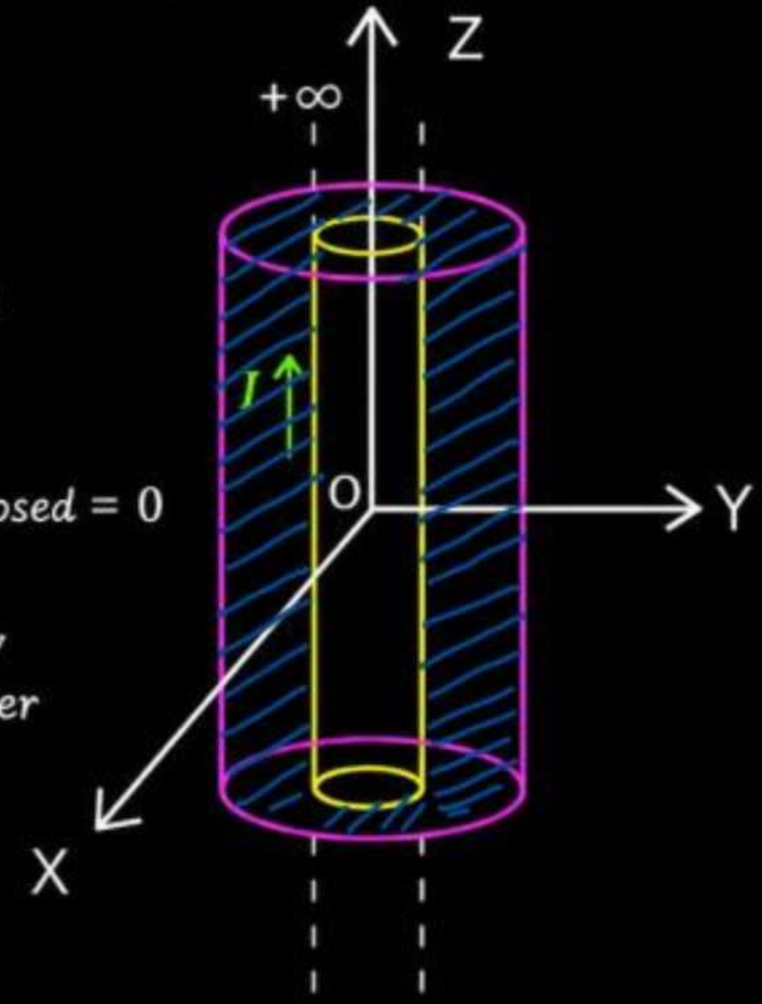
$$\oint_{C_1} \vec{H} . d\vec{l} = I_{enclosed}$$

$$H \oint_{C_1} dl = 0$$

$$H \int_0^{2\pi} \rho \, d\phi = 0$$

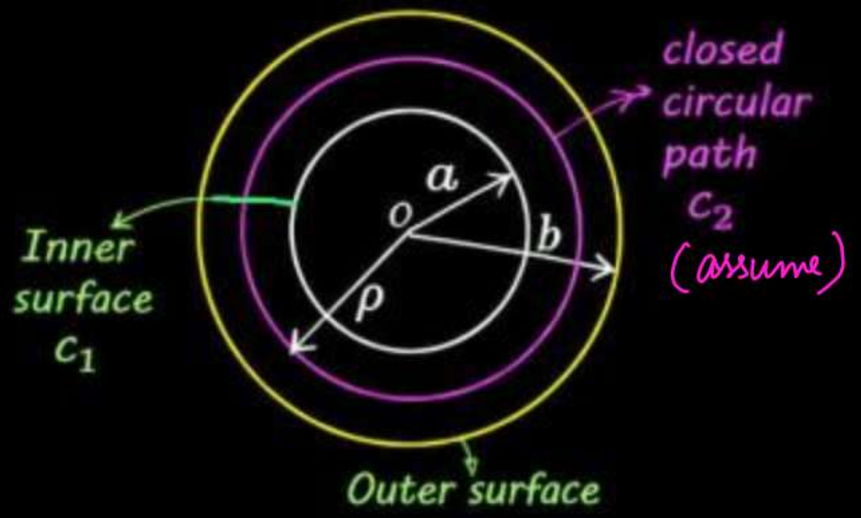
$I_{enclosed} = 0$ ↓ ∵ Hollow cylinder

$$H = 0$$



∴ Inside the cylinder, magnetic field intensity is zero. -∞

(ii) For $a \le \rho \le b$



Applying Ampere's Circuital Law,

$$\oint_{c_2} \vec{H}_2 \cdot d\vec{l} = I_{enclosed}$$

To calculate $I_{enclosed}$,

$I_{enclosed}$ = current per unit area × area of closed surface c_2

$$I_{enclosed} = \frac{I}{\pi(b^2-a^2)} \times \pi(\rho^2-a^2)$$

$\therefore$

$$\oint_{c_2} \vec{H} \cdot d\vec{l} = \frac{I(\rho^2-a^2)}{(b^2-a^2)}$$

$$H \oint_{c_2} d\vec{l} = \frac{I(\rho^2-a^2)}{(b^2-a^2)}$$

$$H \quad (2\pi\rho) = \frac{I(\rho^2-a^2)}{(b^2-a^2)}$$

$$\boxed{H = \frac{I}{2\pi\rho}\left[\frac{(\rho^2-a^2)}{(b^2-a^2)}\right]}$$

A/m

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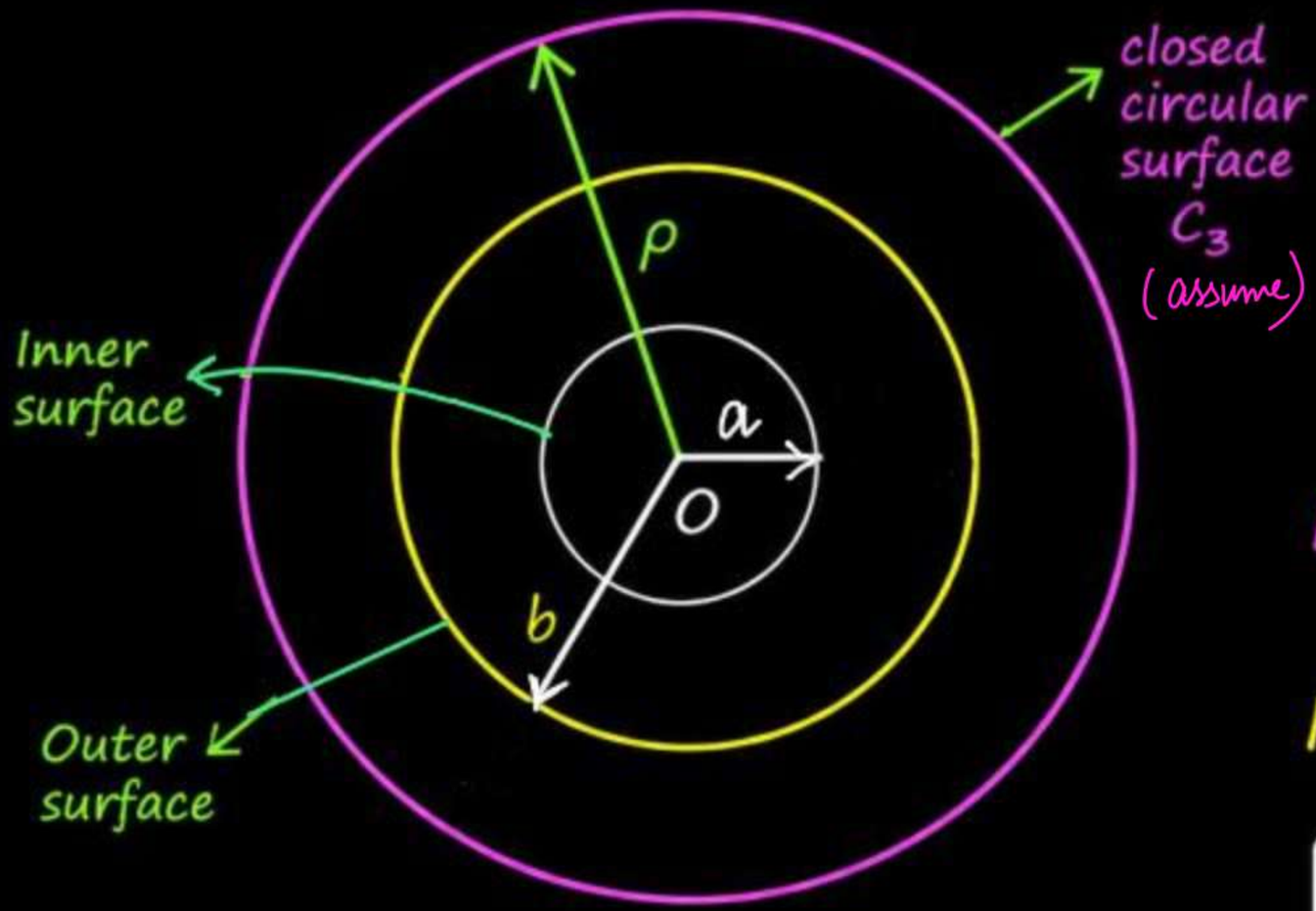
(ii) For $\rho \geq b$



Applying Ampere's Circuital Law,

$$\oint_{C_3} \vec{H} \cdot d\vec{l} = I_{enclosed}$$

$$H \oint_{C_3} dl = I$$

$$H \quad (2\pi\rho) = I$$

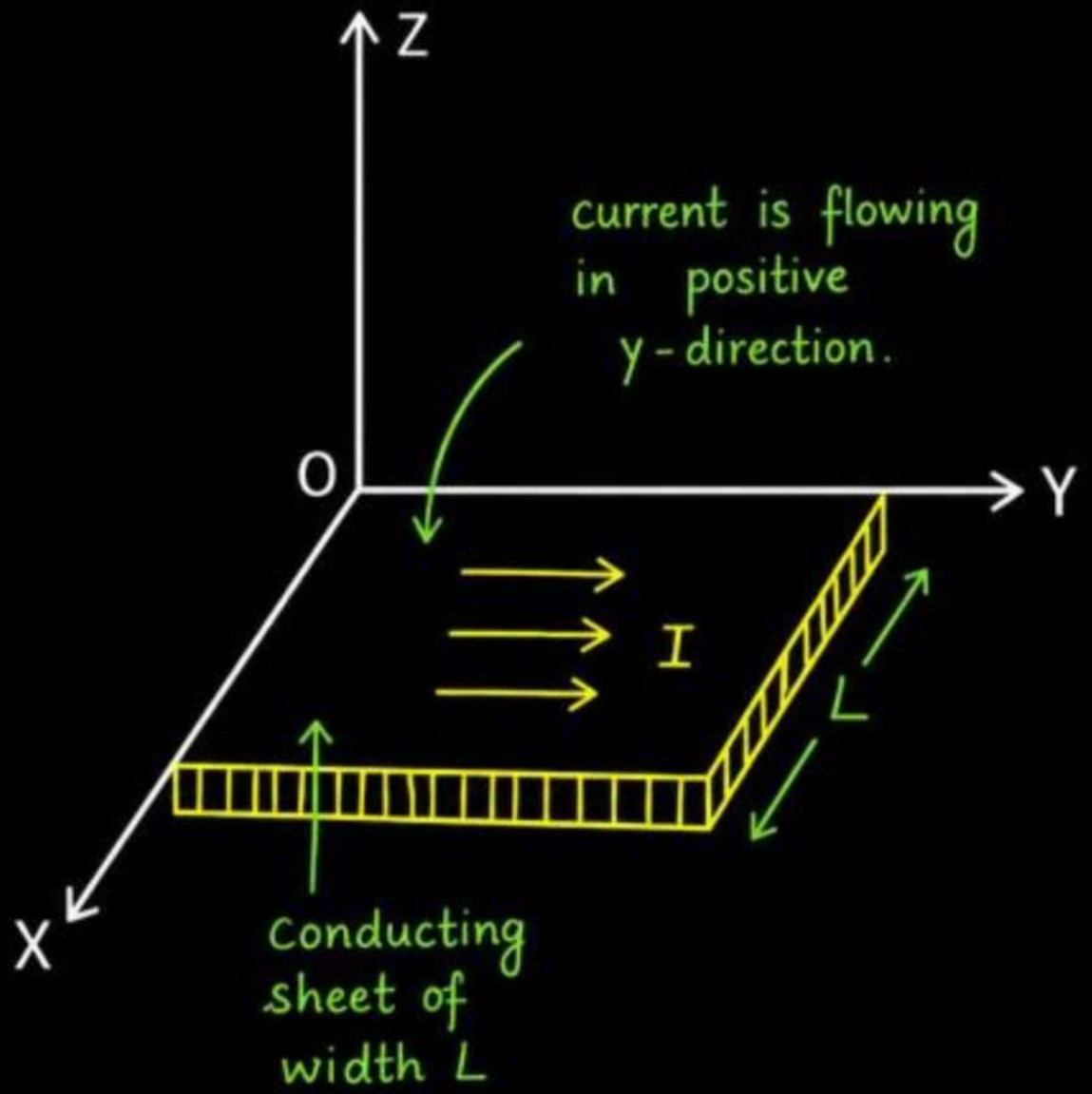
$$H = \frac{I}{2\pi\rho} \quad A/m$$

MAGNETIC FIELD INTENSITY DUE TO UNIFORM SHEET OF CHARGE

Line charge density K,

$$K = \left(\frac{I}{L}\right) \quad A/m$$

$$\therefore \quad I = KL$$



The components of $\vec{H}$ along z axis are zero ,

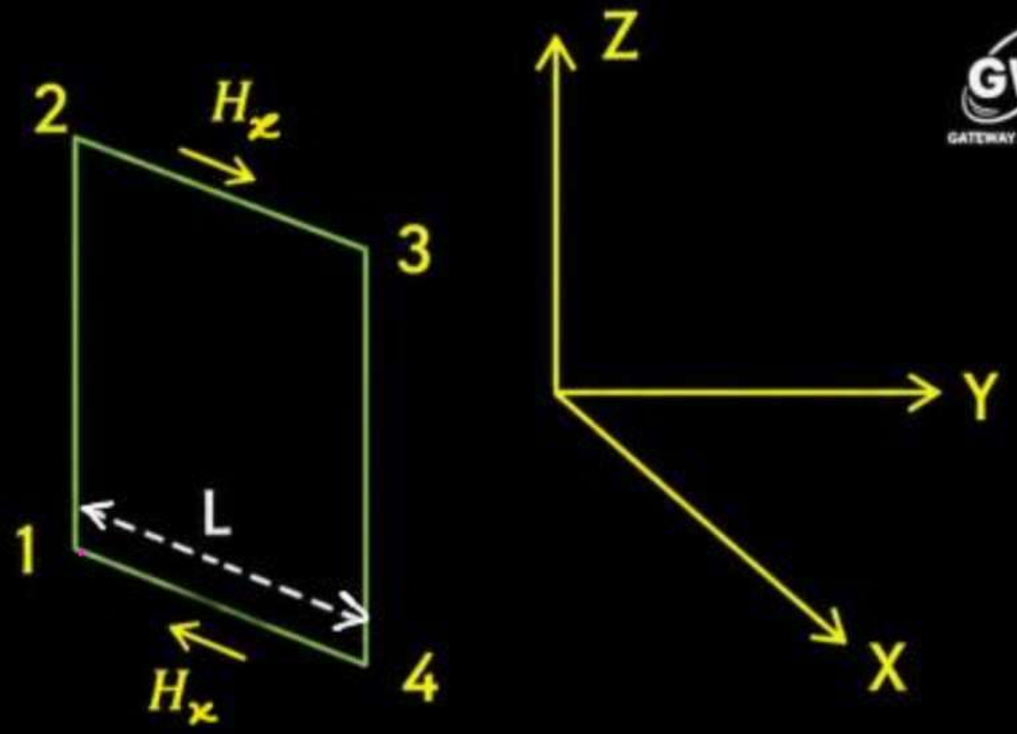
components of $\vec{H}$ along $1 \rightarrow 2$ and 3 and 4 are zero.

$$\oint \vec{H} \cdot d\vec{l} = \int_1^2 \vec{H} \cdot d\vec{l} + \int_2^3 \vec{H} \cdot d\vec{l} + \int_3^4 \vec{H} \cdot d\vec{l} + \int_4^1 \vec{H} \cdot d\vec{l} = I_{enclosed}$$

$$0 + H_x L + 0 + H_x L = KL$$

$$2 H_x = K$$

$$\boxed{H_x = \frac{K}{2}}$$



In Vector Form,

$$\boxed{\vec{H} = \frac{1}{2} K \hat{a}_N}$$

Unit vector $\hat{a}_N$ normal to current sheet

A REVIEW OF BASIC LAWS

(1) $$\oint \vec{\mathbf{E}} \cdot \vec{dS} = \left(\frac{q}{\varepsilon_0}\right) \quad \text{or} \quad \oint \vec{\mathbf{D}} \cdot \vec{dS} = q$$

This is **Gauss's law of electrostatics.** This law states tnat the electric flux through a closed surface is equal to the net charge enclosed by the surface divided by the permittivity constant ε_0.

(2) $$\oint \vec{\mathbf{B}} \cdot \vec{dS} = 0$$

This is **Gauss's law for magnetism.** This law states that the magnetic flux through a closed surface is zero.

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(3) $$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

This is **Faraday's law of electromagnetic induction**. This law states than an electric field is produced by changing magnetic field.

(4) $$\oint \vec{\mathbf{B}} \cdot \vec{dl} = \mu_0 I \quad \text{or} \quad \oint \vec{\mathbf{H}} \cdot \vec{dl} = I$$

This is **Ampere's law for magnetic field** due to steady current. This law states that the amount of workdone in carrying a unit magnetic pole one around a closed arbitrary path linked with the current is μ_0 times the current I.

Que) Why the closed surface integration of magnetic flux density is zero and of electric flux density is equal to flux?

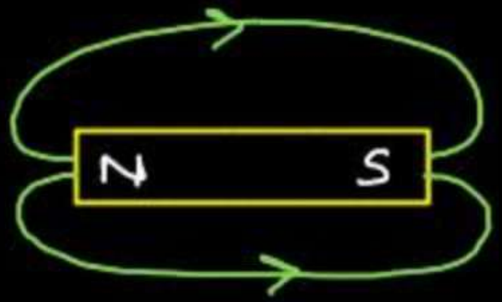
1. Magnetic flux density (Gauss's Law for Magnetism)

$$\oint \vec{B} \cdot d\vec{S} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Reason: Magnetic monopoles do not exist.

Magnetic field lines always form closed loops (north–south pairs).



So, the net magnetic flux through any closed surface is zero.

$$\Psi_{closed} = 0 \quad \text{(always)}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

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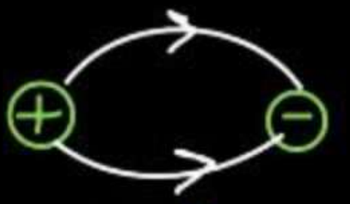
2. Electric Flux Density (Gauss's Law)

$$\oint_s \vec{E} \cdot \vec{dS} = \left(\frac{q}{\varepsilon_0}\right) \quad \text{or} \quad \oint_s \vec{D} \cdot \vec{dS} = q$$



$\phi_{closed} = Q_{enclosed} = q$ (always)

$\therefore \oint \vec{D} \cdot \vec{dS} = q$

Reason: Electric field lines start on positive charges and end on negative charges.

Thus, the closed surface integral equals the charge enclosed, which is the electric flux.

AKTU B.Tech Sem-3

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit -3 : Lec -3

Topics to be Covered

- MAXWELL'S EQUATIONS
- PYQs

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- M.Tech (Honours)
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MAXWELL'S EQUATIONS

AKTU PYQ-2023-24,2022-23,2019-20

INTEGRAL FORM

(1)The total flux coming out of a closed surface is equal to the net charge enclosed.

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \quad \text{or} \quad \oint_S \vec{D} \cdot d\vec{S} = q$$

(2) The surface integral of magnetic flux density over a closed surface is zero.

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

(3) The net e.m.f. induced in a closed path is equal to the surface integral of negative time rate of change of flux density over the surface bounded by closed path.

$$\oint_L \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\int_s \frac{\partial \vec{B}}{\partial t} d\vec{S}$$

(Faraday's Law)

MAXWELL'S EQUATIONS

(4)The total mmf around any closed path must be equal to the surface integral of conduction and displacement densities over the surface bounded by the closed path.

$$\oint_L \vec{B}.d\vec{l} = \int_S \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \cdot d\vec{S}$$

$\vec{B} = \mu_0 H$

$\vec{D} = \epsilon_0 \vec{E}$

$$\oint_L \vec{H}.d\vec{l} = \int_S \vec{J}.d\vec{S} + \int_S \frac{\partial \vec{D}}{\partial t} d\vec{S} \qquad I = \int_S \vec{J}.d\vec{S}$$

$$\boxed{\oint_L \vec{H}.d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) d\vec{S}}$$

MAXWELL'S EQUATIONS

DIFFERENTIAL FORM (Point Form)

The differential forms (point form) of these equations are given below:

$$\text{div}\,.\vec{E} = \frac{\rho}{\epsilon_0} \quad \text{or} \quad \vec{\nabla}\,.\,\vec{D} = \rho$$

$$\text{div}\,.\vec{B} = 0 \quad \text{or} \quad \vec{\nabla}\,.\,\vec{B} = 0$$

$$\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{or} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl}\,\vec{B} = \mu_0\left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \quad \text{or} \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

DERIVATION OF MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM

1. Maxwell's first equation : $\vec{\nabla} \cdot \vec{D} = \rho$

Gauss Law E , Divergence Theorem

According to Gauss's law

$$\oint_s \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

$$q = \int_v \rho \, dV$$

$$\oint_s \vec{E} \cdot \vec{dS} = \frac{1}{\epsilon_0} \int_v \rho \, dV$$

$$\epsilon_0 \oint_s \vec{E} \cdot \vec{dS} = \int_v \rho \, dV$$

$$\oint_s \vec{D} \cdot \vec{dS} = \int_v \rho \, dV \qquad (\epsilon_0 \vec{E} = \vec{D})$$

$$\oint_S \vec{D} \cdot d\vec{S} = \int_v \rho \, dV$$

According to Divergence Theorem ,

$$\oint_S \vec{D} \cdot d\vec{S} = \int_v \vec{\nabla} \cdot \vec{D} \, dV$$

$$\int_v \vec{\nabla} \cdot \vec{D} \, dV = \int_v \rho \, dV$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\text{div } \vec{E} = \frac{\rho}{\epsilon_0}$$

DERIVATION OF MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM

2. Maxwell's second equation : $\vec{\nabla} \cdot \vec{B} = 0$

Gauss Law B, Divergence Theorem

According to Gauss's law for magnetism

$$\oint \vec{B} \cdot d\vec{S} = 0$$

Physical Significance :-It states that magnetic monopoles don't exist, means magnetic field lines always form closed loops.

Transforming the surface integral into volume integral by Gauss's divergence theorem, we have

$$\oint_s \vec{B} \cdot d\vec{S} = \int_v (\vec{\nabla} \cdot \vec{B})\, dV$$

$$\int_v (\vec{\nabla} \cdot \vec{B})\, dV = 0$$

As the volume is arbitrary , the integral must be zero. Hence ,

$$\vec{\nabla} \cdot \vec{B} = 0$$

or

$$\text{div}\, \vec{B} = 0$$

DERIVATION OF MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM

3. Maxwell's third equation : $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$

Faraday's Law , Stoke's Theorem

According to Faraday's law ,

$$\oint_L \vec{E}.d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{\partial}{\partial t}\int_s \vec{B}\, d\vec{S}$$

or $$\oint_L \vec{E}.d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\, d\vec{S}$$

Applying Stoke's Theorem,

$$\oint_L \vec{E}.d\vec{l} = \int_s (\vec{\nabla} \times \vec{E})\, d\vec{S}$$

So, $$\int_s (\vec{\nabla} \times \vec{E})\, d\vec{S} = -\int \frac{\partial B}{\partial t}\, d\vec{S}$$

This eq is true for all surfaces , therefore $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

or $\text{curl}\ \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

DERIVATION OF MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM

4. Maxwell' fourth equation

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

AKTU PYQ-2022-23

Ampere's Circuital Law , Stoke's Theorem

According to Ampere's circuital law, the line integral of magnetic field intensity over a closed path is equal to the total current enclosed by the path

$$\oint_L \vec{H}.d\vec{l} = I = i_c + i_d$$

where I is the total current.

The total current is the sum of conduction current i_c and displacement current i_d, i.e $I = i_c + i_d$

In terms of conduction current density $\vec{J}_c$ and displacement current density $\vec{J}_d$, i_c and i_d can be expressed as

$$i_c = \int_s \vec{J}_c \, d\vec{S} \quad \text{and} \quad i_d = \int_s \vec{J}_d \, d\vec{S}$$

Substituting these values in above eq.

$$\oint_L \vec{H}.d\vec{l} = \int_s \vec{J}_c \, d\vec{S} + \int_s \frac{\partial \vec{D}}{\partial t} d\vec{S}$$

(as $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$ where D is Electric Flux density)

$$\oint_L \vec{H}.d\vec{l} = \int_S \vec{J}\, d\vec{S} + \int_S \frac{\partial \vec{B}}{\partial t} d\vec{S}$$

(as Normally conduction current density $\vec{J}c$ is represented by $\vec{J}$)

Therefore this eq. can be expressed as

$$\oint_L \vec{H}.d\vec{l} = \int_S (\vec{J} + \frac{\partial \vec{B}}{\partial t}) \, d\vec{S}$$

According to Stoke's theorem ,

$$\oint_L \vec{H}.d\vec{l} = \int_S (\vec{\nabla} \times \vec{H}) \, d\vec{S}$$

Therefore

$$\int_S (\vec{\nabla} \times \vec{H}) d\vec{S} = \int_S (\vec{J} + \frac{\partial \vec{B}}{\partial t}) \, d\vec{S}$$

Taking derivatives of both the sides,

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}}$$

AKTU B.Tech Sem-3

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit – 3: Lec - 4

GATEWAY CLASSES

Topics to be Covered

- MAGNETIC SCALAR AND VECTOR POTENTIAL
- MAGNETIC BOUNDARY CONDITIONS
- PYQs

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- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

MAGNETIC SCALAR AND VECTOR POTENTIAL

ϕ_m $\vec{A}$

AKTU PYQ-2022-23,2021-22

Comparison of Magnetic Scalar vs. Vector Potentials

Feature	Magnetic Scalar Potential ϕ_m	Magnetic Vector Potential ($\vec{A}$)
Definition	$\vec{B} = -\vec{\nabla}\phi_m$	$\vec{B} = \vec{\nabla} \times \vec{A}$
Applicability	Valid only in regions without free currents (J = 0)	Valid in any region, including where currents flow
Equation Type	Reduces to Laplace's equation in current-free regions	Generally coupled with Maxwell's equations; requires gauge choice
Usefulness	Simpler for problems like magnetostatics in current-free zones (e.g., outside solenoids)	More general; essential for problems involving currents, EM waves, and quantum mechanics

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{I}\, d\vec{l}}{R}$$

GATEWAY CLASSES

Proof of $\vec{B} = \vec{\nabla} \times \vec{A}$

AKTU PYQ-2022-23

GATEWAY CLASSES

1. Gauss Law for magnetism

Maxwell's equation states that the divergence of the magnetic field (B) is zero, which reflects the physical fact that magnetic monopoles do not exist:

$$\nabla \cdot \mathbf{B} = 0$$

2. Apply a Vector Calculus Identity

From vector mathematics, the divergence of the curl of *any* continuously differentiable vector field $\mathbf{V}$ is always identically zero:

$$\nabla \cdot (\nabla \times \mathbf{V}) = 0$$

Here $V = \vec{A}$

3. Compare and Deduce

By comparing step 1 and step 2, we can see that since the divergence of both $\mathbf{B}$ and $\nabla \times \mathbf{A}$ is zero, $\mathbf{B}$ must be proportional to the curl of some vector field. We define this vector field as the magnetic vector potential $\mathbf{A}$:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Que) When vector magnetic potential is given by

$$\vec{A} = \frac{1}{r^2}\left(2\cos\theta\ \hat{a}_r + \sin\theta\ \hat{a}_\theta\right)$$

find the magnetic flux density.

Magnetic flux density $\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$

In spherical co-ordinate system,

$$\vec{B} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\ \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & r\cdot A_\theta & r\sin\theta\ A_\phi \end{vmatrix}$$

$$\vec{B} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\,\hat{a}_\theta & r\sin\theta\ \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{2\cos\theta}{r^2} & \frac{\sin\theta}{r} & 0 \end{vmatrix}$$

$\frac{\sin\theta}{r} \rightarrow r A_\theta$

$$\vec{B} = \hat{a}_r(0-0) - r\hat{a}_\theta(0-0) + \frac{1}{r}\hat{a}_\phi\left[\frac{\partial}{\partial r}\left(\frac{\sin\theta}{r}\right) - \frac{\partial}{\partial\theta}\left(\frac{2\cos\theta}{r^2}\right)\right]$$

$$= \frac{1}{r}\hat{a}_\phi\left[-\frac{\sin\theta}{r^2} + \frac{2\sin\theta}{r^2}\right]$$

$$= \frac{\sin\theta}{r^3}\hat{a}_\phi$$ Ans.

GW GATEWAY CLASSES

AKTU PYQ-2025-26

Que) Given the magnetic vector potential,

$$\vec{A} = -\frac{\rho^2}{4}\,\hat{a}_z\ (Wb/m)$$

Calculate the total magnetic flux crossing

$$\phi = \frac{\pi}{2},\ 1 \le \rho \le 2\ m,\ 0 \le z \le 5\ m$$

$\vec{A}$ ($\vec{B}$) ψ or ϕ

Magnetic Flux $\psi = \int_S \vec{B} : d\vec{S}$ where

Magnetic flux density $\vec{B} = \vec{\nabla} \times \vec{A}$

In cylindrical co-ordinate system,

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \frac{1}{\rho}\,\hat{a}_\rho & \hat{a}_\phi & \frac{1}{\rho}\,\hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\vec{B} = \begin{vmatrix} \frac{1}{\rho}\,\hat{a}_\rho & \hat{a}_\phi & \frac{1}{\rho}\,\hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & -\frac{\rho^2}{4} \end{vmatrix}$$

$$= -\hat{a}_\phi \left[-\frac{1}{4}(2\rho)\right] = \frac{\rho}{2}\,\hat{a}_\phi$$

Ans

GATEWAY CLASSES

Now, $\psi = \int \vec{B} \cdot d\vec{S}$

where, $\vec{B} = \frac{\rho}{2} \cdot \hat{a}_\phi$

$d\vec{S} = d\rho\, dz\, \hat{a}_\phi$

$\psi = \iint \frac{\rho}{2}\, d\rho\, dz\, \hat{a}_\phi \hat{a}_\phi$ $\hat{a}_\phi \hat{a}_\phi = 1$

$= \frac{1}{2} \int_0^2 \int_1^5 \rho\, d\rho\, dz$

$\psi = \frac{1}{2} \int_1^2 \rho\, d\rho \int_0^5 dz$

$\psi = \frac{1}{2} \left[\frac{\rho^2}{2}\right]_1^2 \left[z\right]_0^5$

$= \frac{1}{4} \left[4 - 1\right] \left[5 - 0\right]$

$= \frac{1}{4} \times 3 \times 5$

$= \frac{15}{4}$ wb Ans.

Que) If the magnetic field $\vec{H} = 100 \sin\theta \, \hat{a}_\theta$ A/m in spherical coordinates, determine $\vec{J}$ at $(10, \pi/2, 0)$.

H_θ

$$\vec{J} = \vec{\nabla} \times \vec{H}$$

$$\vec{J} = \vec{\nabla} \times \vec{H} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\,H_\phi \end{vmatrix}$$

$$\vec{J} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 100\, r \sin\theta & 0 \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \, r \sin\theta \, \hat{a}_\phi \left[100 \sin\theta \right]$$

$$= \frac{100 \sin\theta}{r} \, \hat{a}_\phi$$

At $(r = 10, \theta = \pi/2, \phi = 0)$

$$\vec{J} = \frac{100 \sin(\pi/2)}{10} \, \hat{a}_\phi = 10 \, \hat{a}_\phi \text{ A/m}$$

Ans

GATEWAY CLASSES

Que) Find out the vector magnetic potential due to the following:-

i) Line Current

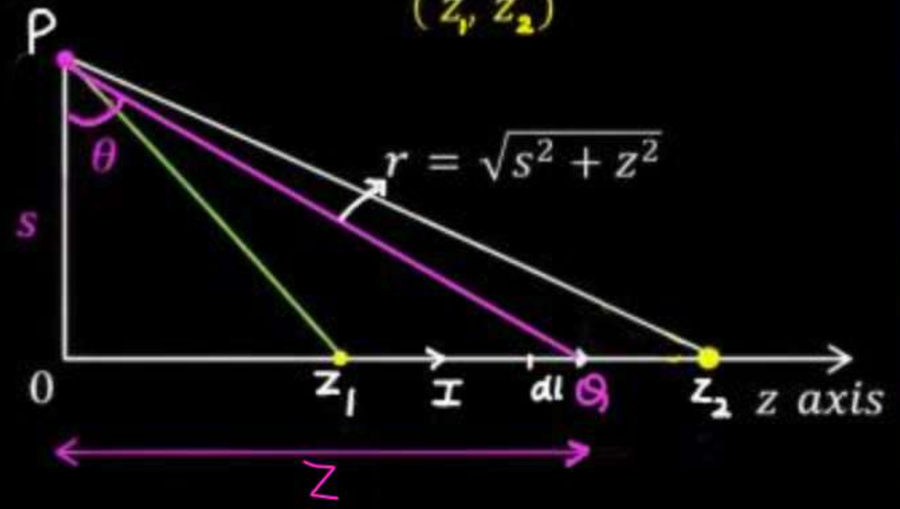
ii) Circular Current Loop

GATEWAY CLASSES

(i) Vector magnetic potential due to Line Current

Let the conductor is *along z axis*

(z_1, z_2)



$$OP = s$$

$$OQ = z$$

$$\therefore \quad PQ = r = \sqrt{s^2 + z^2}$$

$$\vec{A} = \frac{1}{4\pi} \int \frac{\mu_0 I \, d\vec{l}}{r}$$

$$d\vec{l} = dz \; \hat{a}_z$$

$$= \frac{1}{4\pi} \int \frac{\mu_0 I \; dz \; \hat{a}_z}{\sqrt{s^2 + z^2}}$$

$$= \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{dz}{\sqrt{s^2 + z^2}} \; \hat{a}_z \quad -(1)$$

From fig.,

$$\tan\theta = \frac{z}{s}$$

$$\therefore \quad z = s \tan\theta$$

$$dz = s \sec^2\theta \, d\theta$$

Note:- Don't change limits $\because z \to \theta \to z$

Put in (1)

$$\vec{A} = \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{s \sec^2\theta \, d\theta}{\sqrt{s^2 + s^2 \tan^2\theta}} \; \hat{a}_z$$

$$= \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{s \sec^2\theta \, d\theta}{s \sec\theta} \; \hat{a}_z$$

$$= \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \sec\theta \, d\theta \; \hat{a}_z$$

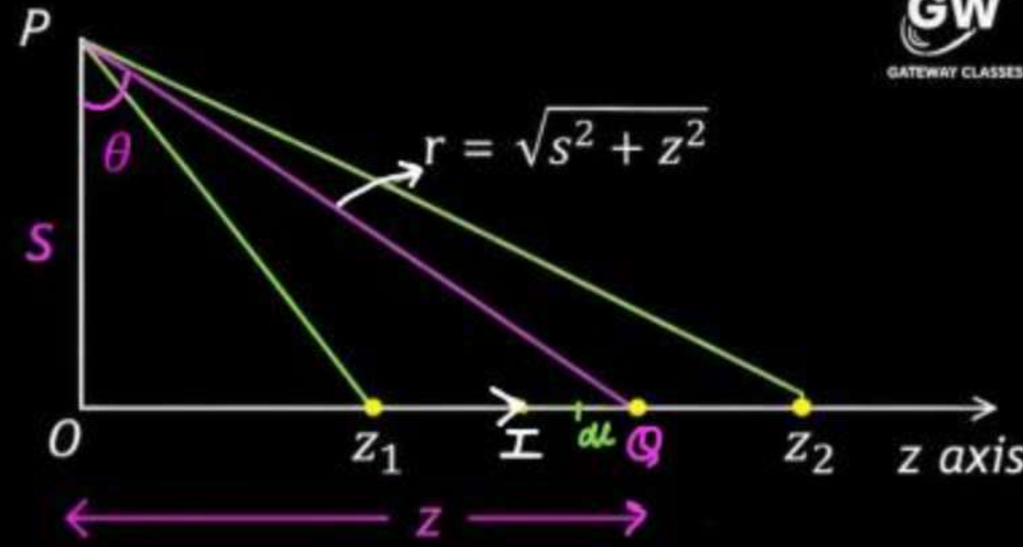
$$\vec{A} = \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \sec\theta \, d\theta \; \hat{a}_z$$

$$= \frac{\mu_0 I}{4\pi} \left[\ln(\sec\theta + \tan\theta) \right]_{z_1}^{z_2} \hat{a}_z \quad \text{(by formula)}$$

Put value of $\sec\theta$ & $\tan\theta$ from fig.

$$= \frac{\mu_0 I}{4\pi} \left[\ln\left(\frac{\sqrt{s^2 + z^2}}{s} + \frac{z}{s} \right) \right]_{z_1}^{z_2} \hat{a}_z$$

$$= \frac{\mu_0 I}{4\pi} \left[\ln\left(\frac{\sqrt{s^2 + z_2^2}}{s} + \frac{z_2}{s} \right) - \ln\left(\frac{\sqrt{s^2 + z_1^2}}{s} + \frac{z_1}{s} \right) \right] \hat{a}_z$$

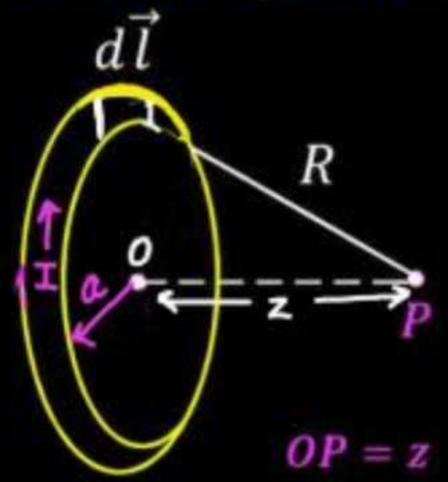


$\therefore$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{\sqrt{s^2 + z_2^2} + z_2}{\sqrt{s^2 + z_1^2} + z_1} \right] \hat{a}_z$$

(ii) Vector magnetic potential due to Circular Current Loop

$$\vec{A} = \frac{\mu_0}{4\pi} \oint \frac{\vec{I} \cdot d\vec{l}}{R}$$



For a circular loop,

$$d\vec{l} = a \, d\phi \, \hat{a}_\phi$$

→ Learn

$$\vec{A} = \frac{\mu_0}{4\pi} \oint \frac{I a \, d\phi}{R} \hat{a}_\phi$$

$$= \frac{\mu_0}{4\pi} \frac{I a}{R} \oint d\phi \, \hat{a}_\phi$$

$$= \frac{\mu_0}{4\pi} \frac{I a}{R} [\phi]_0^{2\pi} \hat{a}_\phi$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{I a}{R} [2\pi] \hat{a}_\phi$$

$$\vec{A} = \frac{\mu_0}{2} \frac{I a}{R} \hat{a}_\phi$$

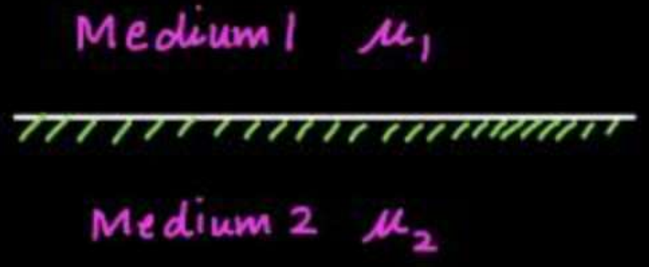
BOUNDARY CONDITIONS FOR MAGNETIC FIELD

AKTU PYQ-2023-24,2022-23,2021-22

In a static medium, the magnetic field is continuous.

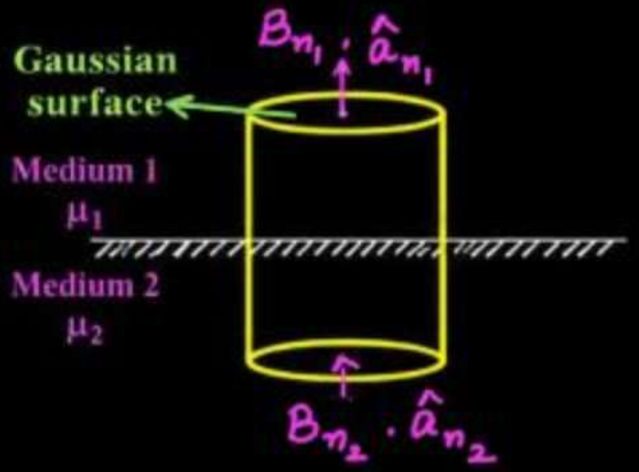
At the boundary between two magnetic materials, the magnetic field may change abruptly both in magnitude and direction.

Consider two magnetically isotropic homogenous materials (medium 1 and medium 2) with permeabilities μ_1 and μ_2 respectively.



(1) Condition for Normal Component of B

Consider a Cylindrical Gaussian surface which lies half in both the media of permeability μ_1 and μ_2. The height of the cylinder is assumed to be small so that the magnetic flux through the curved surface is negligible.



Let B_{n_1} be the average component of B normal to the top surface in medium 1 and B_{n_2} be the average component of B normal to the bottom surface in medium 2.

B_{n_1} is an outward normal while B_{n_2} is inward normal. So, B_{n_1} is positive while B_{n_2} is negative.Let $\hat{a}_{n1}$ and $\hat{a}_{n2}$ be the outward normals to the upper and lower surfaces respectively.

Applying Gauss's law to magnetic field, we have

$$\oint \vec{B} \cdot d\vec{s} = 0$$

For cylindrical surface, we have

$$\oint \vec{B} \cdot d\vec{s} = \Phi_{top}\ \vec{B} \cdot d\vec{s} + \Phi_{bottom}\ \vec{B} \cdot d\vec{s} + \Phi_{side}\ \vec{B} \cdot d\vec{s} = 0$$

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The side of the cylinder is assumed to be very small and hence the contribution of side surface integral is taken as zero, i.e.,

$$\oint_{side} \vec{B} \cdot d\vec{S} = 0$$

$$\text{Therefore } \oint_{top} \vec{B} \cdot d\vec{S} + \oint_{bottom} \vec{B} \cdot d\vec{S} = 0$$

So, we have

$$B_{n_1} \hat{a}_{n_1} \cdot \Delta S - B_{n_2} \hat{a}_{n_2} \cdot \Delta S = 0$$

$$B_{n1} (\vec{\Delta S}) - B_{n_2} (\vec{\Delta S}) = 0$$

Or

$$\boxed{B_{n1} = B_{n_2}} \checkmark$$

Thus, the normal components of magnetic flux densities are equal at the boundary between the two magnetic media, i.e., normal component of $\vec{B}$ is continuous across the boundary between two media.

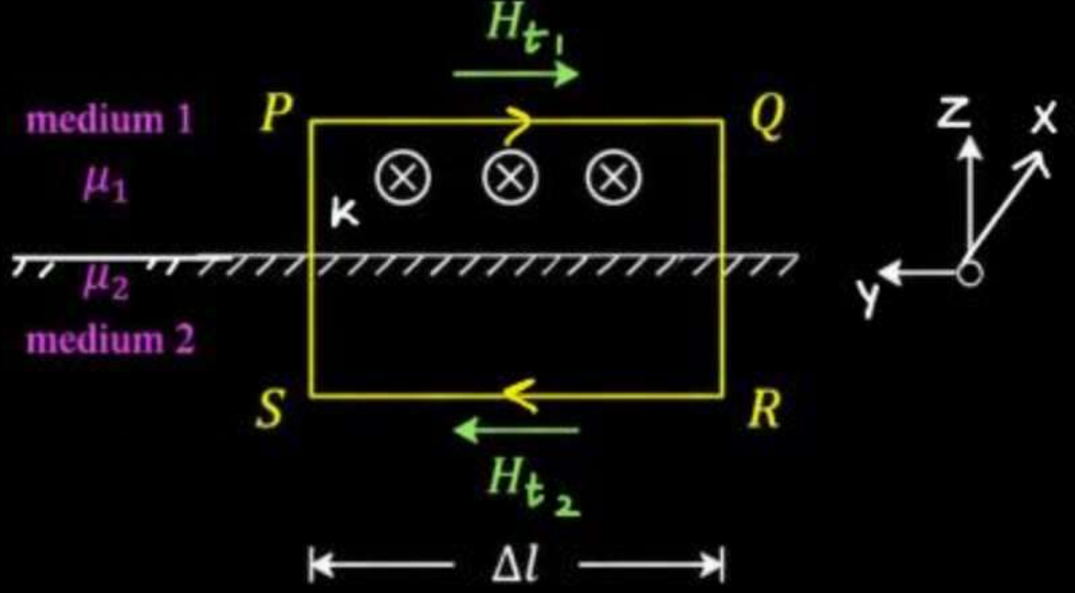
In terms of field intensities, i.e., $\mu_1 H_{n1} = \mu_2 H_{n_2}$

$$\therefore \boxed{H_{n_2} = \frac{\mu_1}{\mu_2} H_{n_1}} \checkmark$$

This shows that the normal component of H is discontinuous by the ratio μ_1/μ_2.

(2) Condition for Tangential Component of H

Let us consider a closed rectangular path PQRS which lies tangential to the interface and close to it.



Applying Ampere's law at the boundary

$$\oint \vec{H} \cdot d\vec{l} = I = k\Delta l$$

At boundary, there exists a sheet current of density k.

For a closed path,

$$H_{t_1}\Delta l - H_{t_2}\Delta l = k\Delta l$$

or

$$H_{t_1} - H_{t_2} = k$$

If the surface current density k zero, then

$$\boxed{H_{t_1} = H_{t_2}}$$

The directions of vectors are specified by the relation

$$(\vec{H}_{t_1} - \vec{H}_{t_2}) = k \cdot \hat{a}_n$$

For tangential components, $\vec{H}_{t_1} - \vec{H}_{t_2} = \vec{k} \times \hat{a}_n$

For tangential B,

$$\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = k$$

Special Case

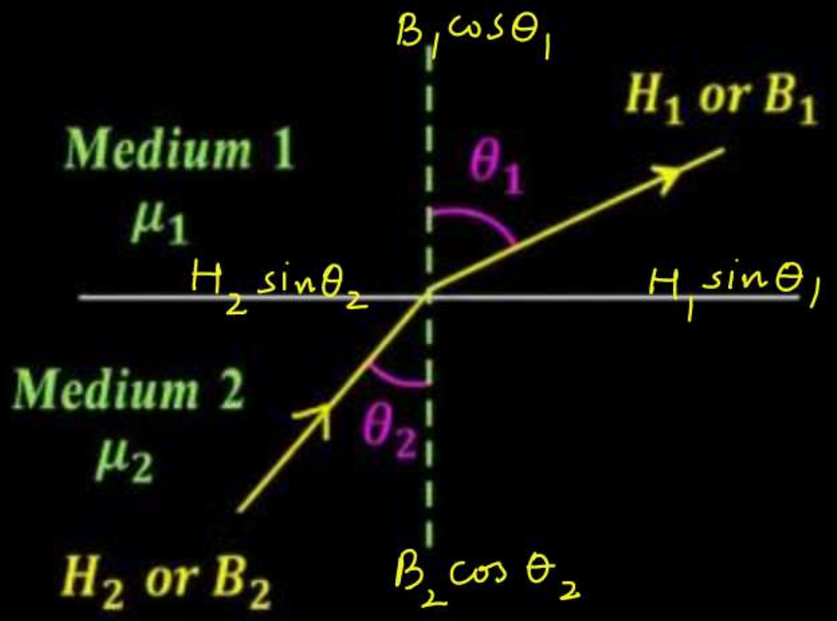
If H_1 (or B_2) and H_2 (or B_2) make angles θ_1 and θ_2 with the normal to the interface between two media as shown in fig., then we have

$$H_{t1} = H_{t2}$$

or $$H_1 \sin\theta_1 = H_2 \sin\theta_2 \quad \text{—(1)}$$

and $$B_{n_1} = B_{n_2}$$

or $$B_1 \cos\theta_1 = B_2 \cos\theta_2 \quad \text{—(2)}$$



Dividing both the equations, we get ① ÷ ②

$$\frac{H_1}{B_1} \tan\theta_1 = \frac{H_2}{B_2} \tan\theta_2$$

or

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{B_1}{B_2}\frac{H_2}{H_1} = \left(\frac{B_1}{H_1}\right)\left(\frac{H_2}{B_2}\right)$$

or

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

This is the **Law of Refraction** for magnetic flux lines at the boundary with no surface current.

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